**Degeneracy:** **Consider the following linear program**

$$\begin{aligned}\min\ & -3x_1 - 4x_2 \\ \text{s.t.}\ & 2x_1 + 3x_2 \le 30 \\ & 3x_1 + x_2 \le 24 \\ & x_2 \le 10 \\ & x_1,\ x_2 \ge 0\end{aligned}$$

**Add slack variables and use these slacks as the initial basis:**

$$\begin{aligned} z &= 0 - 3x_1 - 4x_2 \\ x_3 &= 30 - 2x_1 - 3x_2 \\ x_4 &= 24 - 3x_1 - x_2 \\ x_5 &= 10 - x_2 \end{aligned}$$

**If $x_2$ enters, then $x_3$ and $x_5$ are both candidates to leave. Choosing $x_5$ to leave gives**

$$\begin{aligned} z &= -40 - 3x_1 + 4x_5 \\ x_3 &= 0 - 2x_1 + 3x_5 \\ x_4 &= 14 - 3x_1 + x_5 \\ x_2 &= 10 - x_5 \end{aligned}$$

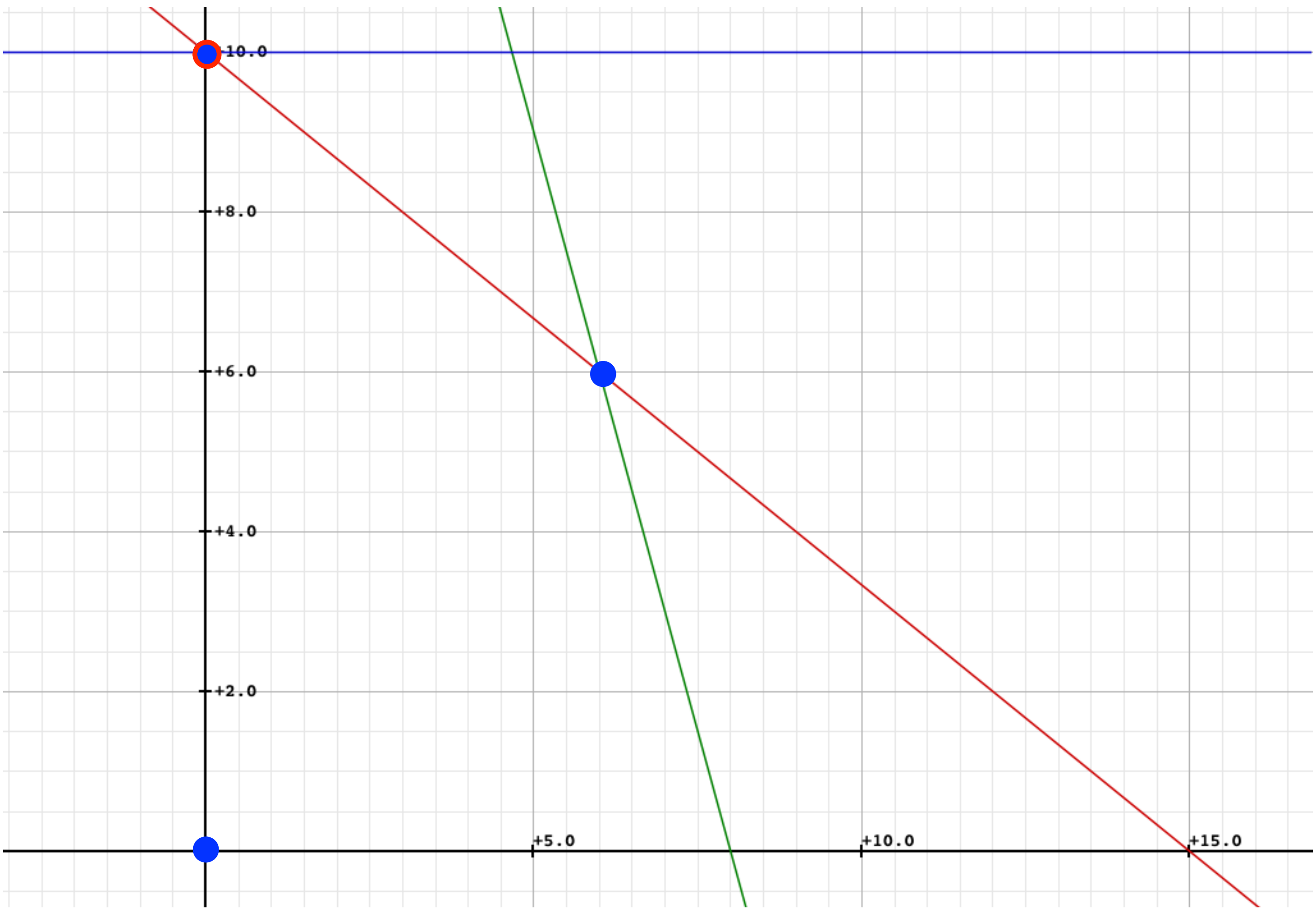
**We have a degenerate basis $\{a_3 , a_4 , a_2\}$.**

**Now $x_1$ enters, and $x_3$ will leave:**

$$\begin{aligned} z &= -40 + 3/2\, x_3 - 1/2\, x_5 \\ x_1 &= 0 - 1/2\, x_3 + 3/2\, x_5 \\ x_4 &= 14 + 3/2\, x_3 - 7/2\, x_5 \\ x_2 &= 10 - x_5 \end{aligned}$$

**We still stay at the same point x = (0, 10, 0, 14, 0) even though the basis changes. At the next Simplex pivot, $x_5$ will enter and $x_4$ will leave, giving x = (6, 6, 0, 0, 4), which turns out to be optimal.**

Geometric Interpretation



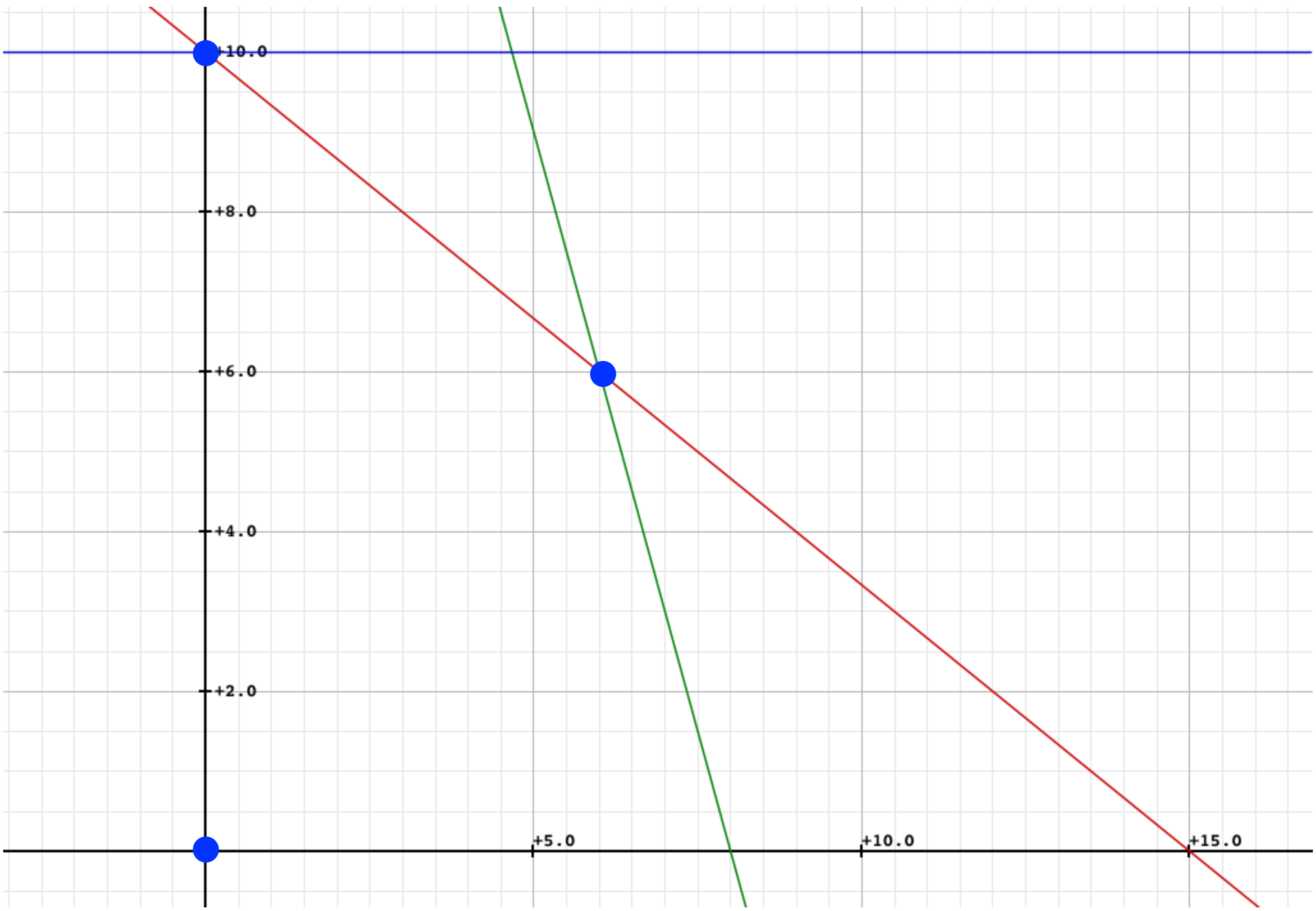

x = (0, 0, 30, 24, 10)

x = (0, 10, 0, 14, 0)

x = (0, 10, 0, 14, 0)

x = (6, 6, 0, 0, 4)

Geometric Interpretation




Suppose we have a problem in inequality form with n variables and m constraints. Degeneracy means accidental intersection of more than n hyperplanes. Thus there will be zero-valued variables in the basis. Several choices of nonbasic variables then define the same exact point.